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Mixing Model for the Calculation of Extinction in Oscillating Flames

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Experiments with opposed flows and flames have shown that the history of the strain from the beginning of a perturbation must be considered in models of turbulent forced flame extinction, and a stochastic model is described to represent the time evolution of the instantaneous scalar dissipation, conditional on the stoichiometric fuel mixture fraction, in a periodically strained mixing layer as a function of frequency and amplitude. The model is based on the concept of a mixinglet, which expresses the scalar quantities within the interface between the opposed jets in terms of an error function. The width of the mixing layer is a function of the sum of the bulk and the turbulent and the periodic strain, and the mixinglet is assumed be randomly convected between the opposed jets. The results reproduce measured trends in the mean and rms of the scalar fluctuations and the dissipation in noncombusting, periodically forced, opposed jet flows and include extinction times in opposed flames that increase exponentially with frequency and decreasing amplitude, again in accord with experiment. The cumulative probability of the scalar dissipation exceeding a critical quenching value is shown to be a near step function of time, which suggests the existence of a competitive mechanism between partial quenching of the reaction and reignition.

Introduction

COMBUSTION oscillations are present in many practical devices and often have undesirable consequences. Thus, the operating range of gas-turbine combustors can be limited by oscillations that may correspond to acoustic frequencies and with amplitudes that are large as a consequence of the energy made available by combustion. On occasion, oscillations can increase heat transfer rates or lead to reduced emissions of oxides of nitrogen,[1] and care is required to ensure that the amplitude of the oscillations does not cause physical damage. In the operation of land-based gas turbines in particular, oscillations can lead to flash back, and it is desirable to determine the extent to which their occurrence can be predicted. One of the mechanisms of these large-amplitude oscillations may be a tendency to extinction, due to a combination of bulk and time-dependent strain, and subsequent reignition, and this is examined here in terms of a model that describes the physical and extinction properties of opposed jets and flames, with imposed oscillations over a range of amplitudes and frequencies.

Previous numerical investigations[2–9] of periodically strained laminar counterflow flames suggest that extinction occurs if the imposed strain exceeds a critical value for sufficient time. Measurements[10] in periodically forced diffusion and premixed turbulent counterflow flames confirmed these approaches by showing that extinction depended on the total duration of pulsation, with timescales that increased quasiexponentially with decreasing amplitude and increasing frequency and that ranged from a few milliseconds to almost a second. Hence, the history of the strain experienced by a flame from the beginning of the perturbation, rather than a single critical value, must be considered in models of turbulent combustion and forced flame extinction. This implies the need for improvements to the extinction criteria,[11–13] which relate local extinction of diffusion flames to values of the scalar dissipation conditional on the stoichiometric mixture fraction exceeding a critical quenching limit χ_q and global extinction to the cumulative probability P_{ext} of conditional dissipation values larger than χ_q.

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This paper presents a stochastic model that describes the time evolution of a passive scalar, such as the fuel mixture fraction, and the conditional dissipation in the periodically strained mixing layer of two opposing turbulent jets, as a function of the bulk strain rate and the frequency and amplitude of the oscillating strain. It stems from the model of Sardi et al.,[14] which considered turbulent flows in the same geometric configuration and in the absence of forced oscillations. The Ref. 14 model has been shown to reproduce measured trends for the evolution of the mean and rms of the scalar and dissipation as a function of the amplitude and frequency of the imposed oscillation in noncombusting turbulent counterflows. The cumulative probability of scalar dissipation values larger than a critical threshold is estimated. Also a criterion for forced flame extinction is proposed that relates the time during which periodic forcing is applied to the temporal evolution of the scalar dissipation and that quantifies the evolution of extinction timescales as a function of frequency and amplitude of the oscillation, in agreement with measurements in combusting counterflows.

Governing Equations

Sardi et al.[14] showed that the instantaneous profile of a passive scalar, for example, the fuel mixture fraction of a diffusion flame $\Theta(z, t)$, can be approximated, at any location z within the mixing layer of two turbulent unforced opposed jets by a mixinglet, expressed in terms of an error function

$$\Theta(z, t) = \tfrac{1}{2}\{1 - \operatorname{erf}[\zeta / w(t)\sqrt{2}]\} \tag{1}$$

where $w(t)$ is the instantaneous width of the mixing layer and z is the sum[15] of the deterministic variable z and a stochastic variable in space, ψ:

$$\zeta = z + \psi \tag{2}$$

The variable ψ was assumed to have a Gaussian probability density function of zero mean and standard deviation s equal to the integral length scale of the turbulent flowfield. Equation (1) is the analytical solution of the transport equation of a passive scalar in homogeneous isotropic turbulence provided that 1) the residence times in the mixing layer were smaller than the eddy turnover time, so that the velocity field was linear, 2) the thickness of the mixing layer was small in comparison to the radii of curvature of the interface, and 3) the distance between neighboring diffusive surfaces was large in comparison to the thickness of the mixing layer.[16] To allow direct comparison with experimental data of the scalar and scalar dissipation fields, constant density has been assumed in the derivation of Eq. (1). A mixinglet profile, similar to that given by Eq. (1), is also obtained in the presence of density gradients.[17]

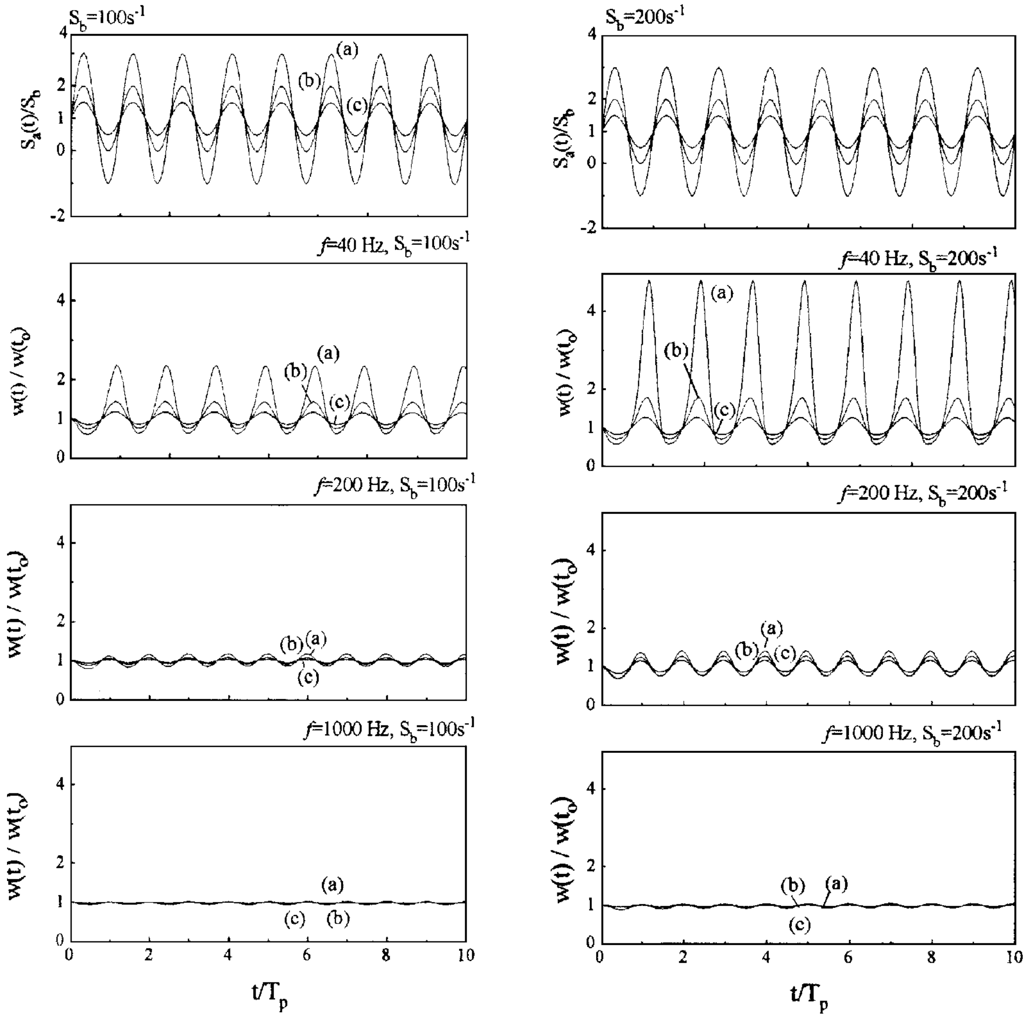

Fig. 1 Time variation of the width $w(t)$ of the periodically forced mixing layer as a function of bulk strain rate and frequency: a, $A/S_b = 0.5$; b, $A/S_b = 1.0$; and c, $A/S_b = 2$.

Substitution of Eq. (1) in the scalar trasport equation leads to[16]

$$\frac{\mathrm{d}[w^2(t)/2]}{\mathrm{d}t} + S_a(t)w^2(t) = D_t \tag{3}$$

so that the instantaneous width $w(t)$ of the mixing layer can be evaluated by solving Eq. (3) with a fourth-order Runge–Kutta method. In Eq. (3) D_t was the molecular diffusion coefficient and $S_a(t)$ the total instantaneous strain acting on an unforced flow defined as

$$S_a(t) = S_b + S_{\mathrm{turb}}(t) \tag{4}$$

with S_b the bulk strain rate equal to the ratio of the bulk velocity U_b to the half-distance $H/2$ of the opposed jets.[18] The instantaneous turbulent strain rate S_{turb} was assumed to vary with a Gaussian probability distribution[19]:

$$P(S_{\mathrm{turb}}) = \frac{1}{\sqrt{2\pi}\,S'_{\mathrm{turb}}}\exp\left[-\left(\frac{S_{\mathrm{turb}} - \bar{S}_{\mathrm{turb}}}{\sqrt{2}S'_{\mathrm{turb}}}\right)^2\right] \tag{5}$$

with mean and rms values being functions of the Kolmogorov timescale t_k:

$$\bar{S}_{\mathrm{turb}} = 0.28/\tau_k, \qquad S'_{\mathrm{turb}} = 0.341/\tau_k \tag{6}$$

and was obtained from direct numerical simulations of turbulent straining on material and propagating surfaces, such as the mixing layer of a diffusion flame and the reaction zone of a premixed flame, in homogeneous and isotropic turbulence.[20,21] The integration time step in Eq. (3) was one Komogorov timescale. Its selection was justified from the simulations of Yeung et al.,[21] who showed that the response times of material and propagating surfaces to changes in the turbulent strain rate and the integral timescale of the turbulent strain were approximately equal to one Kolmogorov timescale.

Hence, each mixinglet given by Eq. (1) was a function of two parameters, the random spatial displacement ψ and the mixing layer width $w(t)$. The time series of the scalar concentration was obtained from the solution of Eq. (3). Scalar moments, e.g., the mean $\langle\Theta\rangle$ and rms θ' of the scalar concentration

$$\langle\Theta(z)\rangle = \frac{1}{N}\sum_{i=1}^{N}\Theta_i(z, t) \tag{7a}$$

$$\theta'(z) = \sqrt{\left[\frac{1}{N}\sum_{i=1}^{N}\Theta_i^2(z, t)\right] - \langle\Theta(z)\rangle^2} \tag{7b}$$

were evaluated by creating a library of mixinglets with each evaluated from Eq. (1) for a different $[\psi, w(t)]$ pair. In Eqs. (7a) and (7b), the subscript i is the scalar value corresponding to the ith $[\psi, w(t)]$ pair, and N is the number of realizations to ensure statistical convergence, here equal to 2.

The instantaneous scalar dissipation χ at any location z within the mixing layer, referred to as unconditional scalar dissipation, and the scalar dissipation conditional on a scalar value $\chi \mid \Theta_{\mathrm{st}}$, e.g., on the stoichiometric mixture fraction Θ_{st} of a diffusion flame, were evaluated with the aid of Eq. (1):

$$\chi(z, t) \equiv 2D_t\left(\frac{\partial\Theta}{\partial z}\right)^2 = \frac{D_t}{\pi w^2(t)}\exp\left[\frac{-z^2}{w^2(t)}\right] \tag{8}$$

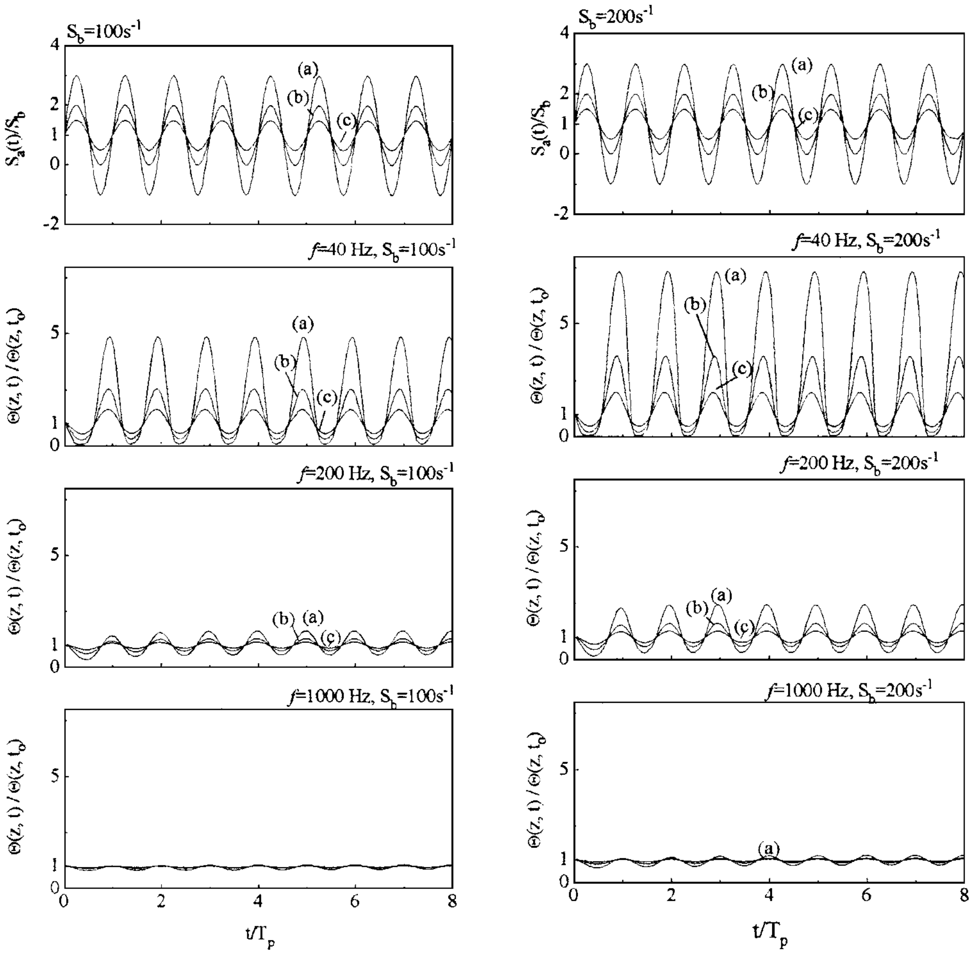

Fig. 2 Time variation of the scalar concentration of the periodically forced mixing layer at $z/(H/2) = -1.3$, where the mean scalar value in the unforced flow was 0.055. Results are functions of bulk strain rate and frequency: a, $A/S_b = 0.5$; b, $A/S_b = 1.0$; and c, $A/S_b = 2$.

$$\chi \mid \Theta_{st}(t) = \frac{D_t}{\pi w^2(t)} \exp\left\{-2\left[\mathrm{erf}^{-1}(2\Theta_{st} - 1)\right]^2\right\} \tag{9}$$

The respective mean, $\langle\chi\rangle$ and $\langle\chi \mid \Theta_{st}\rangle$, and rms, χ' and $\chi' \mid \Theta_{st}$, values were calculated, in analogy with Eqs. (7a) and (7b):

$$\langle\chi(z)\rangle = \frac{1}{N}\sum_{i=1}^{N}\chi_i(z, t) \tag{10a}$$

$$\langle\chi \mid \Theta_{st}\rangle = \frac{1}{N_{st}}\sum_{i=1}^{N_{st}}[\chi_i(t) \mid \Theta_{st}] \tag{10b}$$

$$\chi'(z) = \sqrt{\left[\frac{1}{N}\sum_{i=1}^{N}\chi_i^2(z, t)\right] - \langle\chi(z)\rangle^2} \tag{10c}$$

$$\chi' \mid \Theta_{st} = \sqrt{\left[\frac{1}{N_{st}}\sum_{i=1}^{N_{st}}\chi_i^2(t) \mid \Theta_{st}\right] - \langle\chi \mid \Theta_{st}\rangle^2} \tag{10d}$$

where N_{st} is the number of scalar dissipation values conditional on the stoichiometric mixture fraction.

The model represents the combined effects of molecular and turbulent diffusion, with magnitude fixed by the mixinglet width $w(t)$ and turbulent convection represented by the spatial random displacement. Sardi et al.[14] showed, by comparison with measurements in a noncombusting unforced counterflow, that the model can quantitatively reproduce the single, joint, and conditional statistics of a passive scalar and its dissipation, provided 1) that the residence time within the mixing layer is short in comparison to the eddy turnover time so that there is insufficient time for the turbulence energy to cascade to the smallest eddies and 2) that small-scale characteristics, such as the scalar dissipation, can be locally determined in terms of large energy-containing scales. This last requirement is satisfied in the mixing layer of a counterflow and also at the first diameters downstream the exit of a turbulent jet, which is the stabilization region of many diffusion flames in practical combustors.

The model was extended here to include the effect of a periodic strain rate S_{osc} of amplitude A, frequency f, and angular frequency $\omega = 2\pi f$:

$$S_{osc}(t) = A\sin(\omega t) \tag{11}$$

so that the total instantaneous strain acting on a forced turbulent counterflow increased to

$$S_a(t) = S_b + S_{turb} + S_{osc}(t) \tag{12}$$

The time evolution of the width of the forced mixing layer was obtained from the solution of Eq. (3) with the aid of Eqs. (11) and (12). Results are presented in the next section with the sequence of presentation first considering a single mixinglet of zero random displacement subject to bulk strain at times $t_0 < 0$ and to the sum of bulk and oscillating strain at times $t > 0$. This approach focuses on the time response of the instantaneous mixing layer to periodic forcing and on the resulting scalar concentration and conditional scalar dissipation, as a function of bulk strain and the frequency and amplitude of the oscillating strain rate, so that instantaneous characteristics of the oscillating field, relevant to flame extinction, can be identified. The analysis then proceeds by considering a library of mixinglets, each one randomly displaced in space and subject to the sum of bulk, turbulent, and oscillating strains. The results are discussed in terms of the mean and rms of the scalar and the dissipation

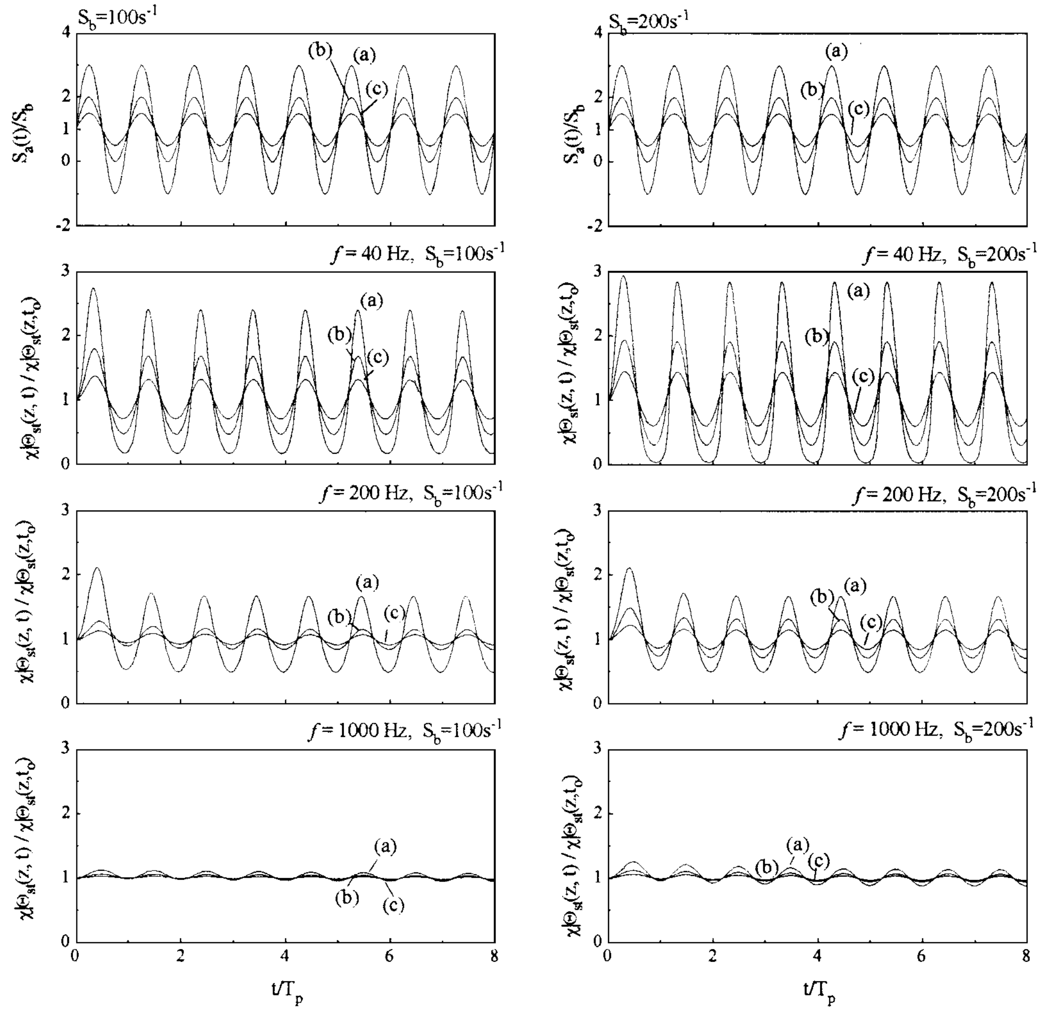

Fig. 3 Time variation of the scalar dissipation conditional on the scalar value of 0.055, which corresponds to the stoichiometric mixture fraction of a CH_4–air flame; results are functions of bulk strain rate and frequency and in all graphs: a, A/S_b = 0.5; b, A/S_b = 1.0; and c, A/S_b = 2.

in conjunction with measurements[22] in noncombusting periodically forced turbulent opposed jets, to demonstrate the extent to which the model represents known flow properties. In all simulations[14] the integral length scale was equal to 2.7 mm, and the Kolmogorov timescale was considered equal to 1 ms.

Knowledge of the time evolution and the statistical moments of a passive scalar and of the conditional scalar dissipation is required in numerical models of turbulent combustion, where the dissipation, conditional on the stoichiometric mixture fraction, is used to correlate to thermal and chemical effects with the degree of stretching of the flame front and global extinction is postulated[11,23] to occur when the cumulative probability of the scalar dissipation exceeds a critical threshold P_q:

$$P_c = 1 - \int_0^{\chi_q} P(\chi \mid \Theta_{\rm st})\, {\rm d}(\chi \mid \Theta_{\rm st}) \geq P_q \qquad (13)$$

with the value of P_q estimated from percolation theory to be of the order of 30%. Percolation theory describes conduction in electrical networks.[24] Peters[11] shows that if local quenching is simulated by interrupted electrical connections within a network then the threshold P_q corresponds to the maximum number of broken connections beyond which no current flows.

Hence, the scalar dissipation distributions obtained by the proposed forced mixinglet model provide insight into the stability of periodically oscillated flames. An underlying assumption in the extinction criterion of Eq. (13) is that global quenching of the reaction occurs within an infinitely small time if 30% of the total instantaneous scalar dissipation at the flame front is higher than the quenching limit. However, measurements[10] showed that periodically forced flames withstood instantaneous strain rates larger that the unforced extinction limits for several pulsation cycles and extinguished only if the duration of the imposed forcing exceeded a critical timescale, which was a function of the amplitude and frequency of the oscillation and the degree of premixedness of the flame. Hence, the extinction criterion of Eq. (13) needs to be modified for calculations of periodically forced flames and may be rewritten as

$$P_{cH} = 1 - \int_0^{\chi_q} P_t(\chi)\, {\rm d}\chi \geq P_q \qquad (14)$$

to include the effects of the duration t_d of the oscillation. In Eq. (14), $P_t(\chi)$ is scalar dissipation probability evaluated within the time interval from $t_0 = 0$, which is the initiation of pulsation, until $(t_0 + t_d)$, and P_q is a critical threshold for global extinction. During pulsation, the flame gradually weakens, and extinction occurs if the duration of the oscillation, t_d, is long enough for P_{cH} to exceed the critical probability P_q. The validity of this analysis is investigated in the next section, where the conditional scalar dissipation time series, evaluated by the mixinglet model, is used to construct the cumulative distribution of Eq. (14) and to evaluate extinction timescales that decrease exponentially with amplitude and increase linearly with frequency of the oscillation, in agreement with measurements.

Results

The instantaneous width of a periodically forced mixing layer, $w(t)$, the scalar concentration $\Theta(z, t)$, and the scalar dissipation $\chi \mid \Theta_{\rm st}$ conditional on a particular scalar value, for example, 0.055, which is the stoichiometric mixture fraction of a CH_4–air diffusion flame, vary sinusoidally with time and with fluctuations that increase

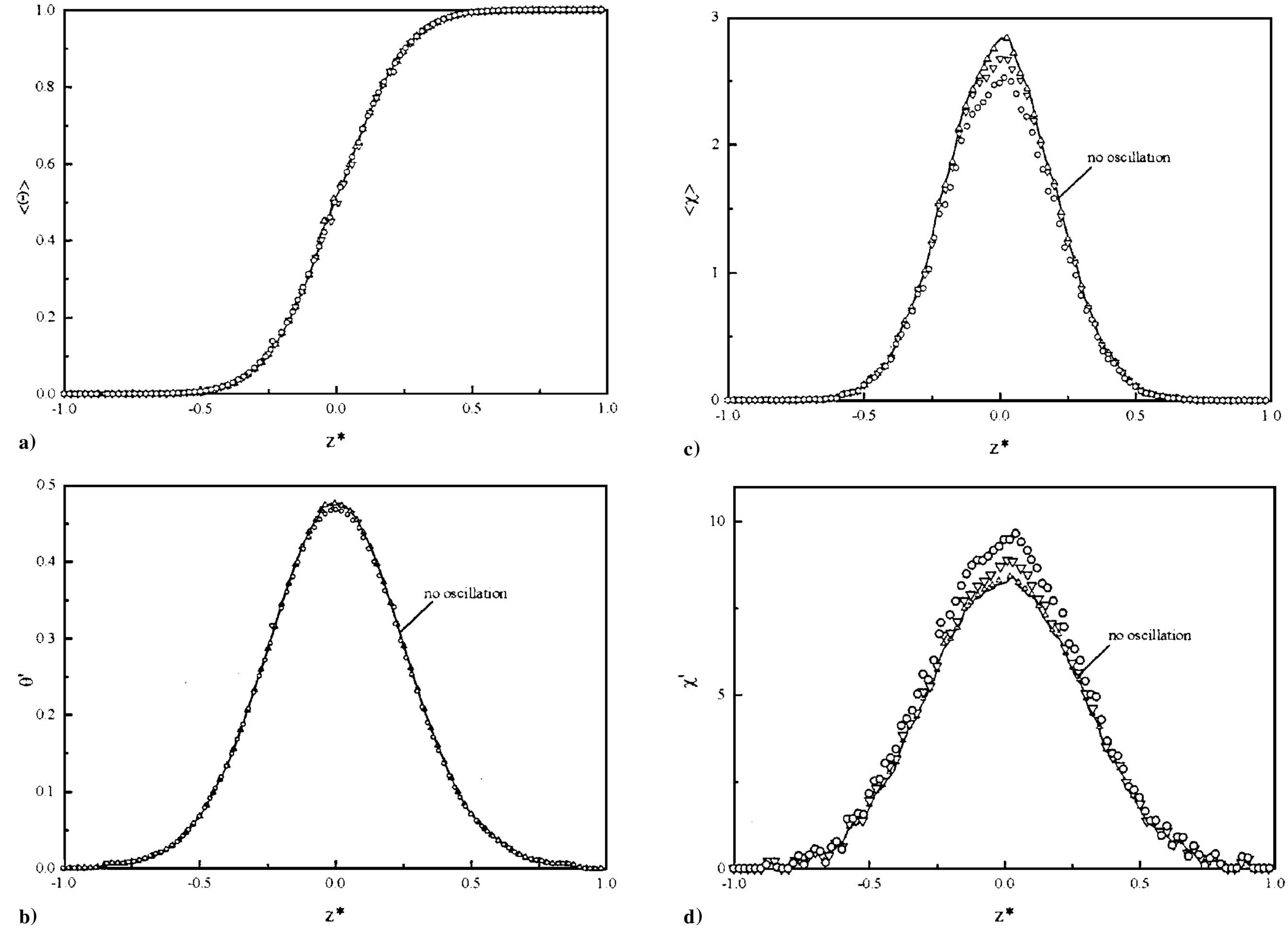

Fig. 4 Mean and rms scalar and mean and rms scalar dissipation as a function of the amplitude of an oscillating strain of frequency of 200 Hz: ——, no oscillation; △, $S_{osc}/S_b = 0.5$; ∇, $S_{osc}/S_b = 1$; and ○, $S_{osc}/S_b = 2$.

with amplitude and decrease with frequency of the oscillating strain. This is shown in Figs. 1–3, where all variables have been normalized with their respective unforced values. For half the oscillation period, instantaneous strain rates $S_a(t)$, larger than the bulk S_b, reduce the thickness of the mixing layer (Fig. 1) and increase the normalized scalar dissipation to values above unity (Fig. 3), implying that, in the presence of combustion, flame extinction is promoted during this part of the cycle. A doubling in the bulk strain, for a constant ratio A/S_b of the amplitude of the oscillating to the bulk strain, leads to a further increase of around 40% in the peak values of the normalized instantaneous dissipation, suggesting that a periodically forced mixing layer, and by extension a diffusion flame, becomes more sensitive to the imposed forcing with increasing S_b. This supports the theoretical analysis of Im et al.,[7,8] who argue that the fluctuations of an oscillating reaction zone can be greatly amplified when the bulk straining is increased and the flame nears extinction.

The temporal evolution of the mixing layer width, i.e., the scalar concentration and the dissipation shown in Figs. 1–3, is asymmetric with respect to the unforced values. The peak protrudes more than the trough in the distribution of the width and vice versa in the distributions of the scalar and the scalar dissipation. This asymmetry increases with decreasing frequency and increasing amplitude of oscillation, as well as with increase in the bulk strain rate, although it can be seen that the scalar dissipation is more asymmetrically distributed than the scalar. Asymmetries in the response of the flame temperature and the heat release rate have been reported in the context of laminar diffusion flame calculations[7,9] and here suggest that periodic forcing is also likely to affect mixing processes in the mean flow at low frequencies.

Accordingly, Figs. 4 and 5 demonstrate that the mean and rms profiles of the scalar and the dissipation, obtained using a library of mixinglets randomly displaced in space and subject to a bulk, a turbulent, and a periodic strain, do not coincide with their respective unforced values, and the differences increase with amplitude. For example, at a frequency of 200 Hz for an oscillating strain twice the bulk value, the rms of the dissipation within the mixing layer increases by approximately 10% (Figs. 4b and 4d), and there is a 10% decrease in the value of the mean scalar dissipation at the stagnation point (Fig. 4c). These findings are in agreement with measurements[22] in the mixing layer of noncombusting turbulent opposed jets forced with pairs of loudspeakers, which revealed that oscillation amplitudes of the order of the bulk velocity in the frequency range of 200–600 Hz lead to an increase in the rms of the dissipation within the mixing layer by factors up to 15% and to a decrease in the mean dissipation by approximately the same factor. Hence, the proposed mixinglet model, which includes the contributions of a bulk, a turbulent, and a periodic strain, can represent the scalar and scalar dissipation moments of a turbulent mixing layer subject to narrow-band unsteady or periodic strain, as can be the case of combusting flows with self-induced or imposed oscillations.

Figure 5 shows that decreasing the oscillation frequency to 40 Hz leads to further reduction in the mean scalar dissipation by approximately 35% and to an increase in the mean width of the mixing layer and the rms of the scalar fluctuations, confirming that low-frequency, large-amplitude oscillations enhance mixing between two streams. On the contrary, increasing the oscillation frequency to 1000 Hz had no effect on the mean mixing field (Fig. 5), as also reported in experiments.[22] To a large extent, this is associated with the damping of the amplitude of the oscillation with increasing frequency, as identified in Figs. 1–3.

Attenuation in the amplitude of temperature and species fluctuations with increasing frequency has been reported in calculations of

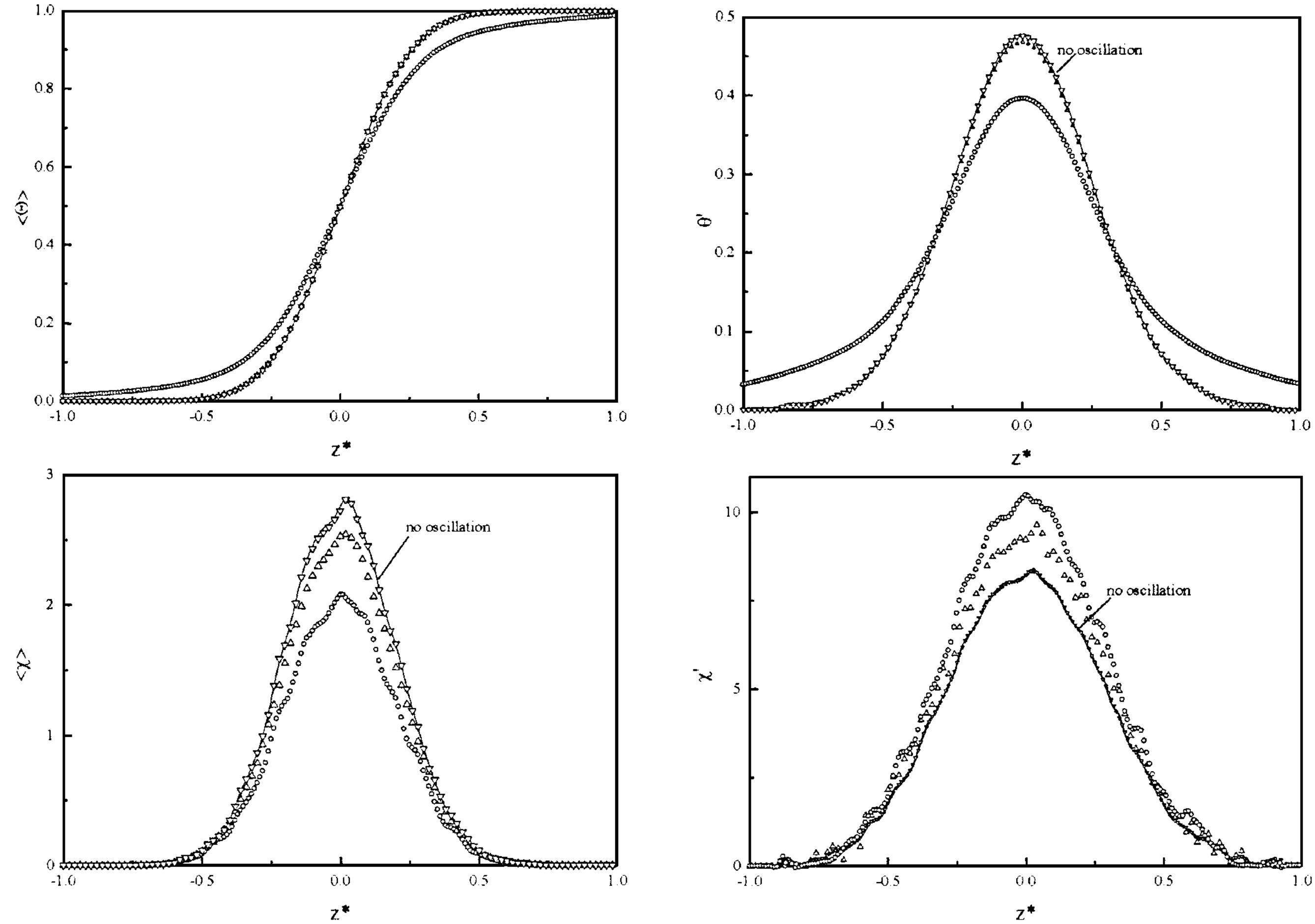

Fig. 5 Mean and rms scalar and mean and rms scalar dissipation as a function of the frequency of an oscillating strain of amplitude $A/S_b = 2$: ——, no oscillation; ○, $f = 40$ Hz; △, $f = 200$ Hz; and ∇, $f = 1000$ Hz.

periodically oscillated laminar flames,[3,8] and it was shown[6,9] that, when the rms of the temperature fluctuations, normalized by its quasisteady value, is plotted as a function of a nondimensional parameter $\eta_{ST} = [(1/2S_b)/(1/\omega)]^{1/2}$, which is the ratio of a diffusion to an oscillation timescale, substantial attenuation occurs at values of n_{ST} of the order of unity. It is interesting to investigate whether a similar scaling applies to the evolution of the scalar dissipation in a turbulent, periodically forced counterflow because this could provide information of the response of a flame to the imposed forcing, as well as an indication of the applicability of extinction criteria for unforced flames.

Figure 6 confirms that the rms of the scalar dissipation that is conditional, e.g., on the stoichiometric mixture fraction of a CH_4–air flame, scales with the nondimensional parameter and attenuation occurs for values of n_{ST} larger than 0.1 with a final cutoff at n_{ST} larger than 10. Thus, the values of n_{ST} can quantify the frequency response of the scalar dissipation to a periodic strain. Smoothing of the scalar gradients occurs when the timescale of the oscillation becomes comparable to the diffusion timescale, implying that flame extinction is less likely to extinguish with increasing the oscillation frequency. Note, however, that the higher the bulk strain rate acting on a forced mixing layer or a flame is, the higher the frequency at which attenuation begins. This explains the larger amplitudes in the scalar dissipation fluctuations identified in Fig. 3 for a doubling in the bulk strain rate. For an oscillation frequency of 40 Hz and a bulk strain rate of $100\,s^{-1}$, the parameter n_{ST} achieves the value of 1.12, and the rms scalar dissipation is attenuated by approximately 65%, as shown by Fig. 6. Doubling the strain rate reduces the value of n_{ST} to 0.79, and the rms of the scalar dissipation is attenuated by only 25% of the maximum value.

For values of the nondimensional parameter n_{ST} lower than about 10, the mixing layer and, by extension, a diffusion flame respond to the oscillation so that the time of pulsation must be consid-

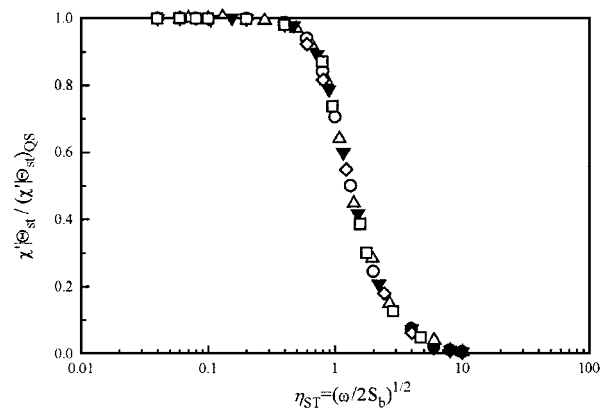

Fig. 6 Variation of the rms conditional scalar dissipation $\chi' \mid \Theta_{st}$, normalized by its quasisteady value $(\chi' \mid \Theta_{st})_{QS}$ as a function of the ratio of the diffusion to the oscillation timescale: ○, $S_b = 100\,s^{-1}$, $A/S_b = 0.2$; △, $S_b = 500\,s^{-1}$, $A/S_b = 0.2$; ▼, $S_b = 200\,s^{-1}$, $A/S_b = 0.2$; ◇, $S_b = 200\,s^{-1}$, $A/S_b = 0.7$; and □, $S_b = 200\,s^{-1}$, $A/S_b = 2$.

ered in models of turbulent combustion and flame extinction. Figures 7a and 8a present the cumulative probability P_{cH} [Eq. (14)] of the values of the scalar dissipation that exceed a quenching limit χ_q. By analogy with measurements,[10] where the extinction strain rates were approximately 5% higher than the bulk values of the forced flames, the critical value of χ_q was considered to be 5% higher than the conditional dissipation at times $t_0 < 0$. Alternatively, the critical dissipation value can be obtained in a more detailed calculation from a laminar flamelet library.[25] It can be seen that P_{cH} varies with time in a steplike distribution with the slope corresponding to time intervals t_{lq}, where $\chi(z, t_0) \mid \Theta_{st}$ exceeds the quenching value

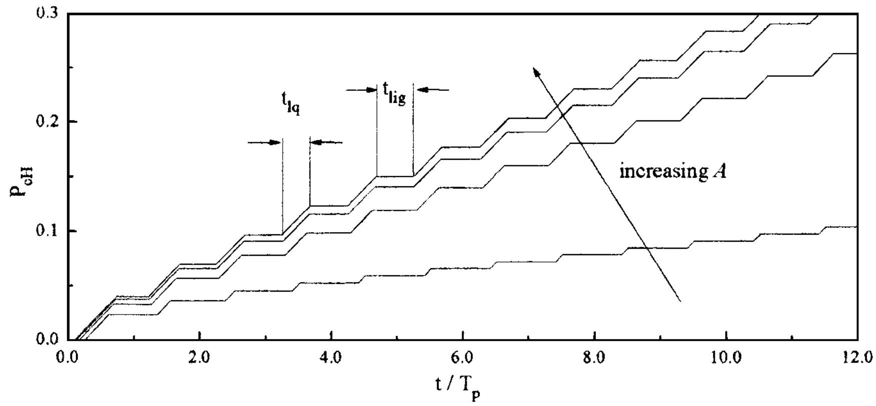

Fig. 7a Cumulative probability, over the total time t_d of pulsation, of values of $\chi(t_0) \mid \Theta_{st}$ higher than a critical quenching value χ_q.

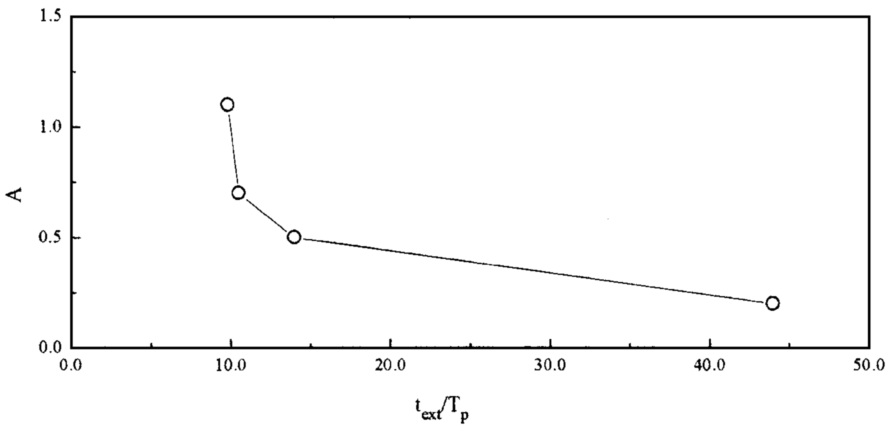

Fig. 7b Extinction time: $A = 1.1$, 0.7, and 0.2.

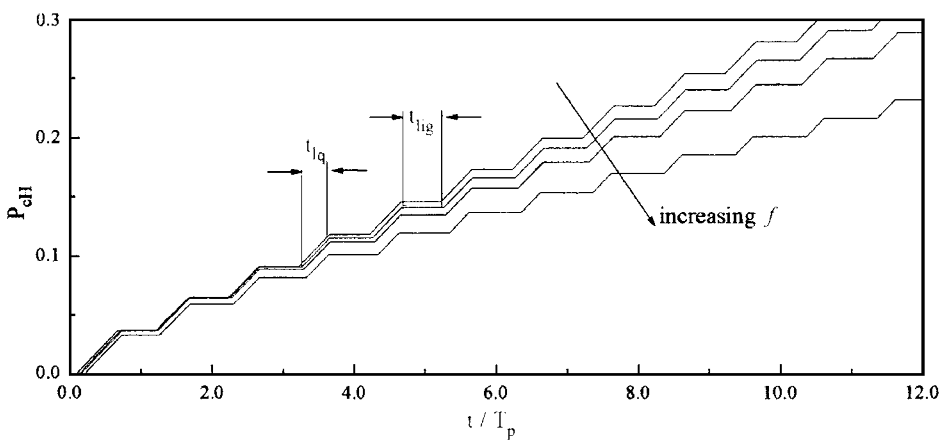

Fig. 8a Cumulative probability, over the total time t_d of pulsation, of values of $\chi(t_0) \mid \Theta_{st}$ higher than a critical quenching value χ_q.

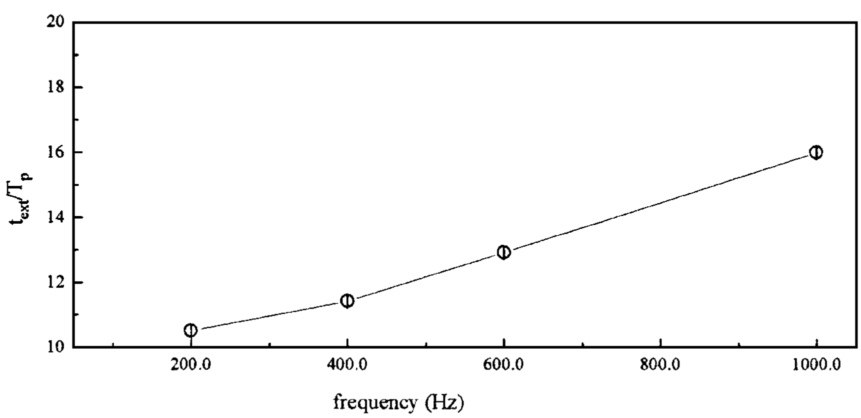

Fig. 8b Extinction time: $f = 100$, 400, and 1000 Hz.

and with the flat part corresponding to time intervals t_{lig}, where $\chi(z, t_0) \mid \Theta_{st}$ is lower to χ_q. During the time interval t_{lq}, partial or full quenching of the reaction takes place due to the instantaneous high values of the scalar dissipation. During the time interval t_{lig}, favorable straining can lead to values of instantaneous conditional dissipation lower than the quenching limit and can cause reignition of the flame due to the high temperatures in the reaction zone. Hence, a competitive mechanism exists between local quenching and reignition. Permanent extinction will occur, according to the extinction criterion proposed in Eq. (7), when the cumulative probability P_{cH} exceeds the critical value. Figures 7b and 8b show extinction timescales, defined as the time of oscillation required for P_{cH}, to exceed the 30% threshold. The number of cycles, t_{ext}/T_p, for extinction increases exponentially with decreasing amplitude in qualitative agreement with measurements in diffusion and premixed periodically oscillated turbulent counterflow flames.[10] The time for which the flame experiences unfavorable straining is longer at lower frequencies, and the scalar dissipation higher leading to faster extinction. This suggests that the extinction time of diffusion flames is a monotonic function of frequency, as shown in Fig. 8b and in agreement with measurements.

Conclusions

The concept of a mixinglet has been extended to describe a turbulent scalar field under periodic variation in the strain rate and has been used to show that, for fluctuations in the width of an instantaneous forced mixing layer, the scalar concentration and the scalar dissipation increase with bulk strain and oscillation amplitude and decrease with frequency.

At low frequencies, the time response of the mixing layer was asymmetric and the asymmetry increased with bulk strain and amplitude, suggesting that periodic forcing is likely to affect mixing processes in the mean flow. Accordingly, the model reproduced measured trends of an increase in the rms of the scalar fluctuations, an increase in the rms of the scalar dissipation, and a decrease in the mean dissipation along the centreline between forced opposed turbulent jets for oscillation amplitudes of the order of the bulk strain rate.

The rms conditional dissipation in the forced flow, normalized with its quasisteady value, scaled with the square root of the ratio of a diffusion to an oscillation timescale, with the former equal to the inverse of twice the bulk strain and the latter equal to the inverse of the angular frequency. This scaling was independent of the bulk strain rate and the amplitude of the oscillating strain so that the ratio of the two timescales can provide an estimate of the response of a mixing layer, and by extension a diffusion flame, to periodic forcing. It was shown that attenuation of the amplitude of the imposed forcing begins at values of the squared timescale ratio of about 0.1, with a final cutoff at values larger than 10.

For values of the timescale ratio below 10, a new extinction criterion was proposed to include the effects of the duration of the oscillation to flame extinction. The results show that the calculated extinction times increase exponentially with decreasing amplitude and increasing frequency in analogy with measurements. The calculated cumulative probability of the scalar dissipation exceeding a critical quenching value was shown to be a near step function of time so that a competitive mechanism exists between the time intervals where the scalar dissipation exceeds the critical value, leading to partial quenching of the reaction, and the times where the scalar dissipation is lower than the quenching limit, resulting in reignition.

Acknowledgments

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